heit folgt. Die Voraussetzung Gl. (3) ist die schwächste heute bekannte Forderung für die Ergodizität, jedoch ist nicht bewiesen, daß sie optimal ist.

Zusammenfassend ergibt sich also, daß man als Bedingungen für ergodisches Verhalten an das Spektrum die Forderung $\bar{\nu}$ nicht groß gegen 1 und an die ergodischen Observablen die Forderung (3) stellt.

Herrn Professor Dr. LUDWIG danke ich für zahlreiche Diskussionen und Anregungen zum Ergodenproblem.

The Motion of Mercury according to the Theory of Thiry and Lichnerowicz

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The extended gravitational theory of THIRY and LICHNEROWICZ [1] is based on the equation

$$G_b{}^a + \varrho\, v^a v_b = 0\,, \tag{1}$$

where $G_b{}^a$ is EINSTEIN's tensor in five dimensions, ϱ a scalar function of matter and v^a a certain "five-velocity". In four dimensions (1) yields EINSTEIN's 10 and MAXWELL's 4 equations, all of them modified by a 15th field function

$$g_{55} = 2\, e^{2\sigma}\,, \tag{2}$$

which is determined by a 15th field equation. For the *static* field outside a spherically symmetric body without an electric charge only 4 of those equations are linearly independent. These yield

$$\left.\begin{aligned} &(e^{\mu})'' = 2\,, \quad (\sigma' e^{\mu})' = (\nu' e^{\mu})' = 0\,,\\ &2\,\mu'' + \mu'^2 + \nu'^2 + 3\,\sigma'^2 = 0\,, \end{aligned}\right\} \tag{3}$$

if the four-dimensional metric is [2] written:

$$\left.\begin{aligned} &\mathrm{d}s^2 = e^{-\sigma}(e^{-\nu}\,\mathrm{d}r^2 + e^{\mu-\nu}\,\mathrm{d}\omega^2 - e^{\nu}\,\mathrm{d}t^2)\\ \text{with}\quad &\mathrm{d}\omega^2 = \mathrm{d}\beta^2 + \cos^2\beta\,\mathrm{d}\varphi^2\,. \end{aligned}\right\} \tag{4}$$

A rigorous solutions of (3) is

$$\left.\begin{aligned} &\nu = \frac{m}{a}\ln\frac{r-a}{r+a}\,, && \sigma = \frac{n}{a}\ln\frac{r-a}{r+a}\,,\\ &\mu = \ln(r^2-a^2)\,, && a^2 = m^2 + \tfrac{3}{4}\,n^2\,, \end{aligned}\right\} \tag{5}$$

with the constants m, n uniquely determined by the central body. Applying the well known formalism of *super-potentials* to the [3] extended theory, we receive

$$m = (\varrho)\,, \qquad 6\,n = 2\,m - (e^{2\sigma} F_{\mu\nu} F^{\mu\nu})\,, \tag{6}$$

where the brackets mean a multiplication by $e^{\sigma}/8\,\pi$ and the three-dimensional *integration over the field-generating body*. If the MAXWELL field $F_{\mu\nu}$ inside this body would be purely radiational, then $F_{\mu\nu} F^{\mu\nu} = \mathfrak{B}^2 - \mathfrak{E}^2 = 0$; but inside the sun we must also assume a COULOMB contribution. Thus we have even

$$F_{\mu\nu} F^{\mu\nu} < 0\,, \quad \text{hence} \quad n > \tfrac{1}{3}\,m\,. \tag{7}$$

Now it is easily [4] seen, that the five-velocity v^a of a *testbody* always describes a five-dimensional geodesic. In the special case of an electrically neutral test-body this also means a geodesic four-velocity (in general this is modified [2] not only by the LORENTZ force, but also by an additional term dependent on the variability of σ). For a motion in the plane $\beta = 0$ of the metric (4) this yields (with D, E = const):

$$(\mathrm{d}r/\mathrm{d}\varphi)^2 = (E^2\, e^{2\mu-2\nu} - e^{-\nu-\sigma})\, D^{-2}\,, \tag{8}$$

giving with (5) in the usual approximation a nearly circular *ellipse* with a slowly *moving perihelion*. The angular velocity of the latter turns out [2] to have EINSTEIN's value multiplied by the factor

$$f = \frac{6\,m+n}{6\,m-3\,n} > \frac{19}{15} > \frac{5}{4}\,, \tag{9}$$

where the numerical estimate followed from (7). Thus we have a relative *deviation above* $\frac{1}{4} = 25\%$, whereas the observations of mercury [5] have confirmed EINSTEIN's expression with an uncertainty *below* 2%.

It must be emphasized that our result has nothing to do with the question, whether the new field variable (2) shall be called a variable [6] gravitational "constant", a dielectricity [7] of the vacuum or something else. To show this *independence of an interpretation* of σ clearly, we have not given it any name at all.

Implicitely the above result is already contained in former [8] papers, where we have considered JORDAN's extended theory of gravitation. There the situation could be saved by a certain condition on the undetermined parameters of the field equations. But the theory of THIRY and LICHNEROWICZ [1] — which was independently developed also by SCHMUTZER [7] — is based on the uniquely arranged equation (1). Thus being a *completely determined special case* of JORDAN's [6] theory, its contradiction with experience cannot be avoided.

The detailed calculations have been performed in 1955 within a research for the Deutsche Forschungs-Gemeinschaft under Prof. LUDWIG.

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